Approximation Algorithms for Traffic Grooming in WDM Rings

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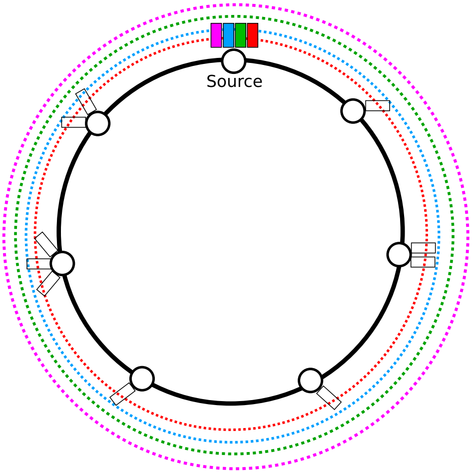
Single-Source WDM Rings



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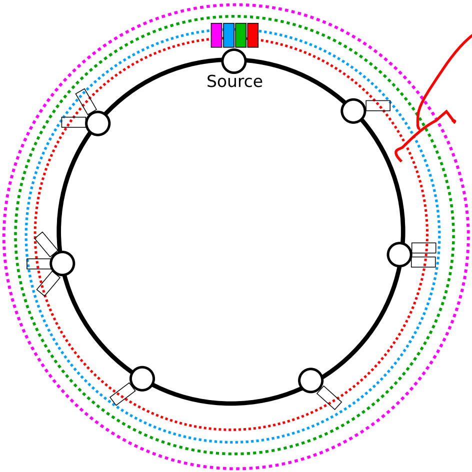

- WDM ring with given set of wavelengths, each with fixed capacity C

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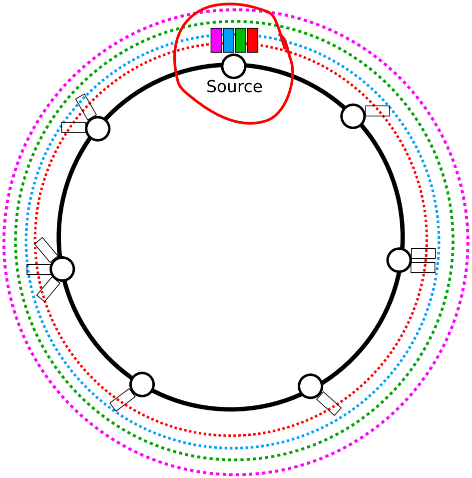

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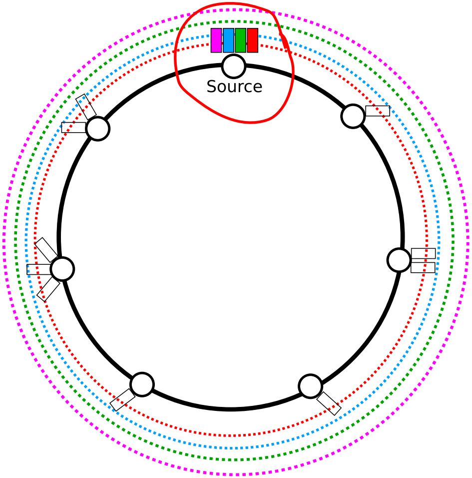

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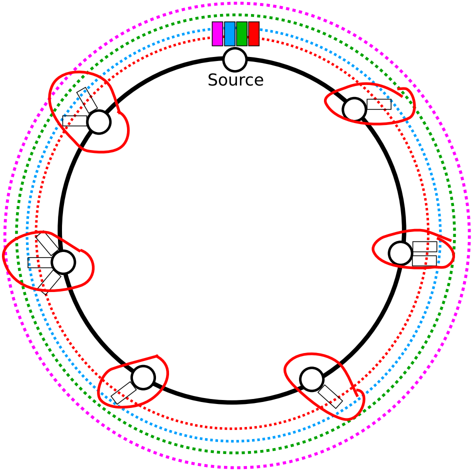

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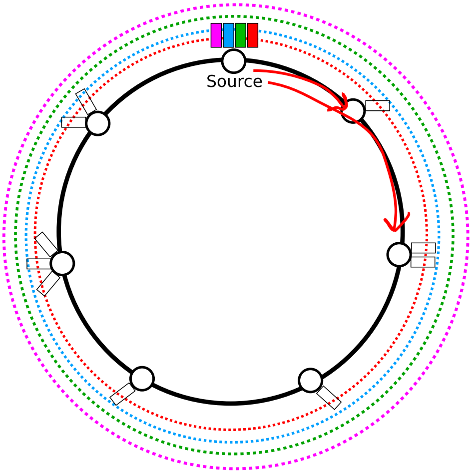

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- Single source/hub from which all other destination nodes receive data
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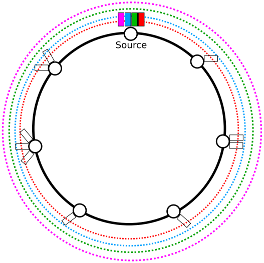
- A path from the source to a destination has a pre-determined route (e.g. all clockwise)

The Tunable Ring Grooming Problem



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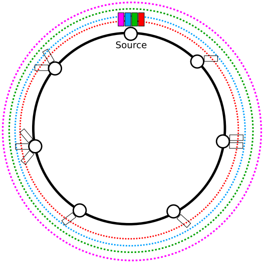

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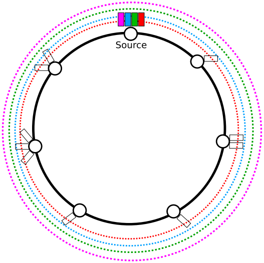

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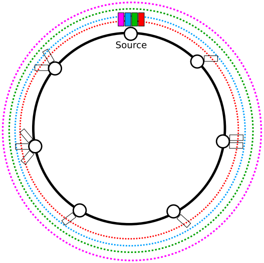

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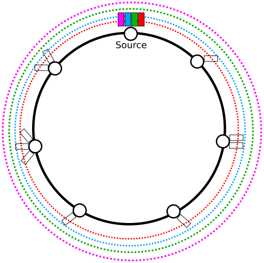

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Sample Instance of the Tunable Ring Grooming Problem

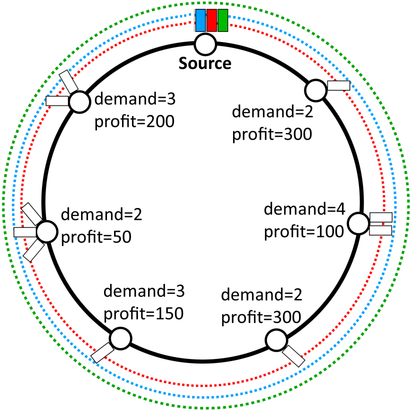

Figure: Capacity $C = 4$ for each wavelength. **Objective:** Tune ADMs and groom requests onto wavelengths to maximize total profit of all satisfied requests.

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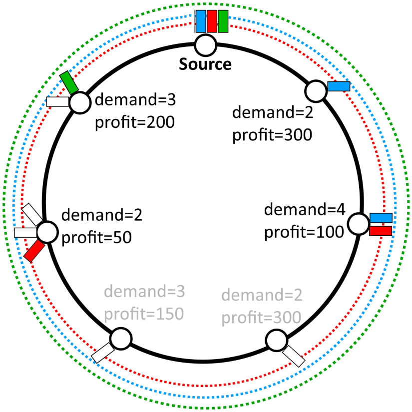

Figure: A solution. Profit = 650. Is it optimal?

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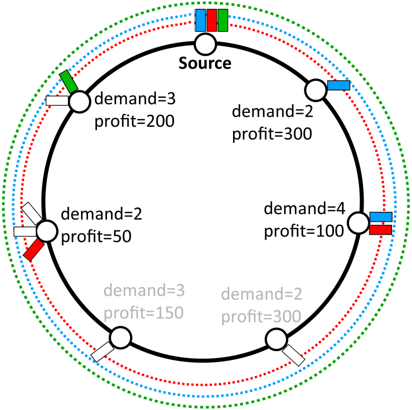

Figure: Profit = 650

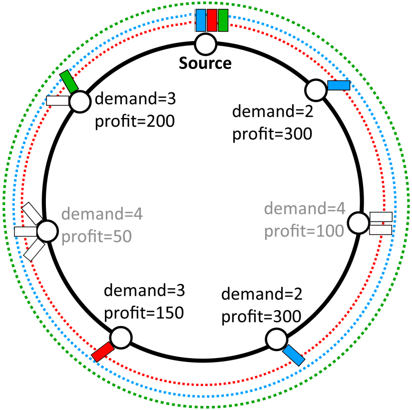

Figure: Profit = 950

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- Polynomial time **approximation schemes** for these special cases
- The "general case" that the number of ADMs is one or more appears to be the most challenging
- New **approximation algorithm** for the general case

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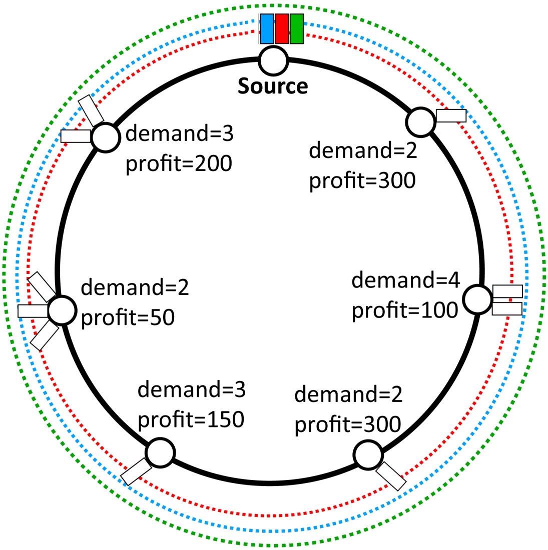
  - If $q = 1$, profit is guaranteed to be within $1/2$ of optimal
  - If $q = 2$, profit is guaranteed to be within $2/3$ of optimal
  - If $q = 10$, profit is guaranteed to be within $10/11$ of optimal

The General Case: The Algorithm



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3. Pack A onto wavelengths with First Fit Decreasing (FFD)
4. **if** *some request in A was not packed* **then**
    5. Let r denote first request not packed by FFD
    6. Let B be the set containing r and all requests which were packed with demand $\geq$ demand(r)
    7. Discard the request with the least profit from B
    8. **if** *r was not discarded* **then**
        9. Pack r in place of the discarded request

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- The running time is $O(R \log R + RW)$ where R is the number of requests and W is the number of wavelengths

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  - Attempts to improve solution using heuristic rules, including splitting
- Experiments using heuristic
  - Heuristic profit divided by optimal profit
  - Optimal found with linear programming

Experimental Results: Parameters

Parameter	Possible values
Wavelength capacity C	4, 8, 16
Number of wavelengths	5
Number of requests	16, 32
Probability α that a request has two ADMs (one ADM otherwise)	$\alpha = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$
Demand limited to fraction $1/q$ of capacity	$q = 1, 2$
Density of request	Constant or variable ($\in U[1/2, 2)$)

Table: Parameters used in generating random instances

Sample Results

- When $q = 1$, approximation algorithm guarantees ratio of $\frac{1}{2}$

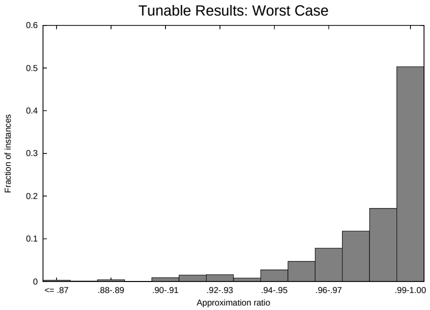

Figure: Worst ratios found in experiments. Parameters: 5 wavelengths, wavelength capacity $C = 16$, $q = 1$, $\frac{1}{2}$ of nodes have 1 ADM and remaining have 2 ADMs

Future Work

- Generalizing to allow requests to demand *more* than a wavelength's capacity
- Tighter approximation bounds
- What if the direction of travel for a request is not pre-determined? Can we still find good approximation algorithms?
- Using splitting in algorithm, not just heuristic