Approximation Algorithms for Traffic Grooming in WDM Rings

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Problem Statement

Single-Source WDM Rings

- WDM ring with given set of wavelengths, each with fixed capacity
- Single source/hub from which all other destination nodes receive data
- Source node can transmit on all wavelengths
- Each destination node has some number of tunable ADMs
- A path from the source to a destination has a pre-determined route (e.g. all clockwise)
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- **Objective**: Tune ADMs and groom requests onto wavelengths to maximize total profit of all satisfied requests.
Sample Instance of the Tunable Ring Grooming Problem

Figure: Capacity $C = 4$ for each wavelength. **Objective:** Tune ADMs and groom requests onto wavelengths to maximize total profit of all satisfied requests.
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**Figure:** A solution. Profit = 650. Is it optimal?
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Figure: Profit = 650

Figure: Profit = 950
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- Polynomial time approximation schemes for these special cases
- The “general case” that the number of ADMs is one or more appears to be the most challenging
- New approximation algorithm for the general case
The General Case

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- **Main Result:** A polynomial time approximation algorithm that guarantees solutions within $\frac{q}{q+1}$ of optimal, i.e.
  - If $q = 1$, profit is guaranteed to be within $1/2$ of optimal
  - If $q = 2$, profit is guaranteed to be within $2/3$ of optimal
  - If $q = 10$, profit is guaranteed to be within $10/11$ of optimal
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2. Let $A = S$ if total demand $\leq CW\frac{q}{q+1}$, otherwise let $A$ be the minimal prefix of $S$ with total demand $> CW\frac{q}{q+1}$

-Pack $A$ onto wavelengths with First Fit Decreasing (FFD)

-if some request in $A$ was not packed

-Let $r$ denote first request not packed by FFD

-Let $B$ be the set containing $r$ and all requests which were packed with demand $\geq$ demand($r$)

-Discard the request with the least profit from $B$

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   7. Discard the request with the least profit from $B$
5. if $r$ was not discarded then
   9. Pack $r$ in place of the discarded request
The General Case: Analysis

- The approximation algorithm is proved correct and analyzed in the paper.
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- The running time is $O(R \log R + RW)$ where $R$ is the number of requests and $W$ is the number of wavelengths.
Heuristics and Experiments

- Heuristic “on top” of approximation algorithm
  - Performs $q/(q + 1)$-approximation algorithm for general case
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  - Performs $q/(q + 1)$-approximation algorithm for general case
  - Attempts to improve solution using heuristic rules, including splitting
- Experiments using heuristic
  - Heuristic profit divided by optimal profit
  - Optimal found with linear programming
Experimental Results: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength capacity $C$</td>
<td>4, 8, 16</td>
</tr>
<tr>
<td>Number of wavelengths</td>
<td>5</td>
</tr>
<tr>
<td>Number of requests</td>
<td>16, 32</td>
</tr>
<tr>
<td>Probability $\alpha$ that a request has two ADMs (one ADM otherwise)</td>
<td>$\alpha = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$</td>
</tr>
<tr>
<td>Demand limited to fraction $1/q$ of capacity</td>
<td>$q = 1, 2$</td>
</tr>
<tr>
<td>Density of request</td>
<td>Constant or variable ($\in U[1/2, 2]$)</td>
</tr>
</tbody>
</table>

Table: Parameters used in generating random instances
Sample Results

- When $q = 1$, approximation algorithm guarantees ratio of $\frac{1}{2}$

Figure: Worst ratios found in experiments. Parameters: 5 wavelengths, wavelength capacity $C = 16$, $q = 1$, $\frac{1}{2}$ of nodes have 1 ADM and remaining have 2 ADMs
Future Work

- Generalizing to allow requests to demand more than a wavelength’s capacity
- Tighter approximation bounds
- What if the direction of travel for a request is not pre-determined? Can we still find good approximation algorithms?
- Using splitting in algorithm, not just heuristic